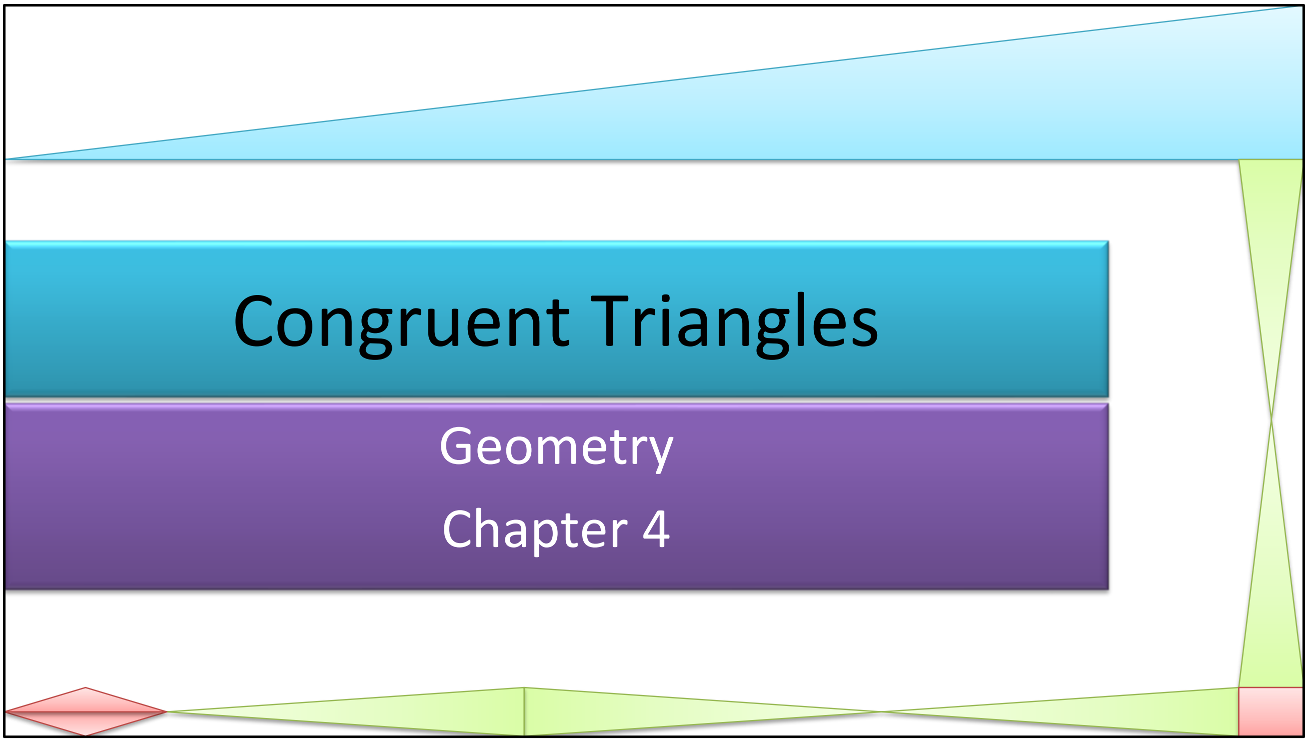

Geometry 4

- This Slideshow was developed to accompany the textbook
  - *Larson Geometry*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
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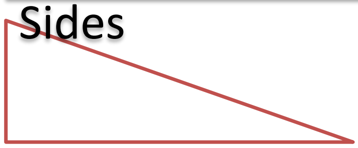
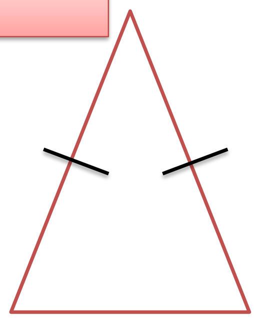
4.1 Apply Triangle Sum Property

Classify Triangles by Sides



Scalene Triangle

No congruent sides

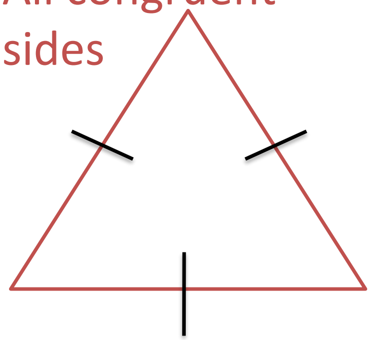


Isosceles Triangle

Two congruent sides

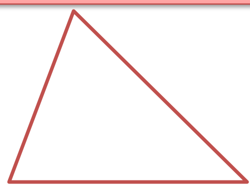
Equilateral Triangle

All congruent sides

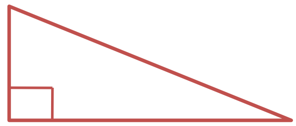


4.1 Apply Triangle Sum Property

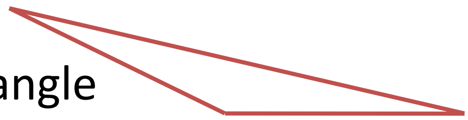
Classify Triangles by Angles



Acute Triangle
3 acute angles

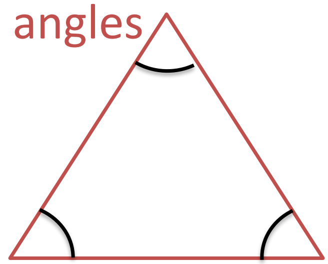


Right Triangle
1 right angle



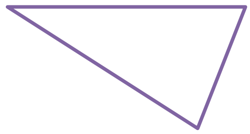
Obtuse Triangle
1 obtuse angle

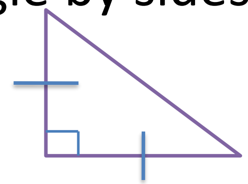
Equiangular Triangle
All congruent angles



4.1 Apply Triangle Sum Property

- Classify the following triangle by sides and angles





Scalene, Acute
Isosceles, Right

4.1 Apply Triangle Sum Property

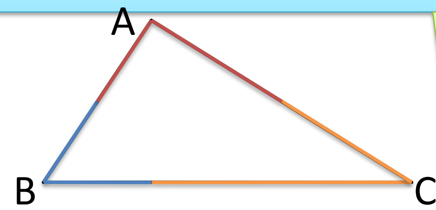
- ΔABC has vertices A(0, 0), B(3, 3), and C(-3, 3). Classify it by is sides. Then determine if it is a right triangle.

Find length of sides using distance formula
AB = $\sqrt{(3 – 0)^2 + (3 – 0)^2} = \sqrt{9 + 9} = \sqrt{18} \approx 4.24$
BC = $\sqrt{(-3 – 3)^2 + (3 – 3)^2} = \sqrt{(-6)^2 + 0} = \sqrt{36} = 6$
AC = $\sqrt{(-3 – 0)^2 + (3 – 0)^2} = \sqrt{9 + 9} = \sqrt{18} \approx 4.24$
Isosceles

Check slopes to find right angles (perpendicular)
$m_{AB} = (3 – 0)/(3 – 0) = 1$
$m_{BC} = (3 – 3)/(-3 – 3) = 0$
$m_{AC} = (3 – 0)/(-3 – 0) = -1$
AB ⊥ AC so it is a right triangle

4.1 Apply Triangle Sum Property

- Take a triangle and tear off two of the angles.
- Move the angles to the 3rd angle.
- What shape do all three angles form?



Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

$m\angle A + m\angle B + m\angle C = 180°$

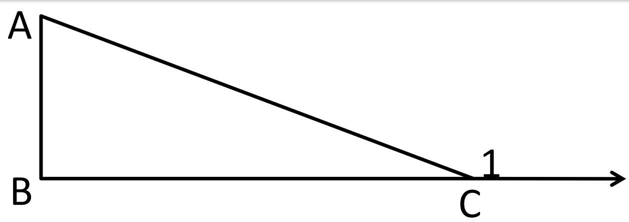
Straight line

4.1 Apply Triangle Sum Property

Exterior Angle Theorem

The measure of an exterior angle of a triangle = the sum of the 2 nonadjacent interior angles.

$m\angle 1 = m\angle A + m\angle B$



Proof:

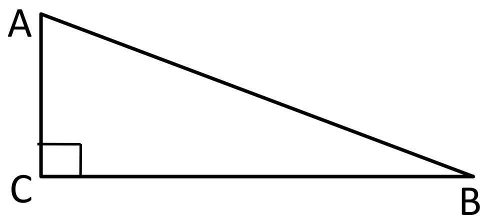
$m\angle A + m\angle B + m\angle ACB = 180°$	(triangle sum theorem)
$m\angle 1 + m\angle ACB = 180°$	(linear pair theorem)
$m\angle 1 + m\angle ACB = m\angle A + m\angle B + m\angle ACB$	(substitution)
$m\angle 1 = m\angle A + m\angle B$	(subtraction)

4.1 Apply Triangle Sum Property

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

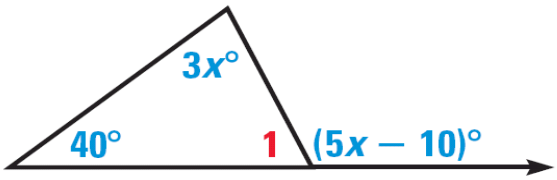
$m\angle A + m\angle B = 90°$



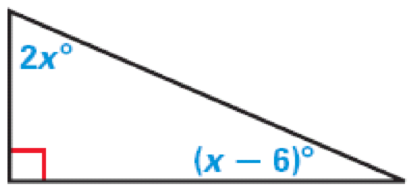
The proof involves saying that all three angles = 180. Since $m\angle C$ is 90, $m\angle A + m\angle B$ = 90.

4.1 Apply Triangle Sum Property

- Find the measure of $\angle 1$ in the diagram.



- Find the measures of the acute angles in the diagram.



$40 + 3x = 5x - 10 \rightarrow 50 = 2x \rightarrow x = 25$
$m\angle 1 + 40 + 3x = 180 \rightarrow m\angle 1 + 40 + 3(25) = 180 \rightarrow m\angle 1 + 40 + 75 = 180 \rightarrow m\angle 1 = 65$

$2x + x - 6 = 90 \rightarrow 3x = 96 \rightarrow x = 32$
Top angle: $2x \rightarrow 2(32) = 64$
Angle at right: $x - 6 \rightarrow 32 - 6 = 26$

4.1 Apply Triangle Sum Property

- *221 #2-36 even, 42-50 even, 54-62 even = 28 total*

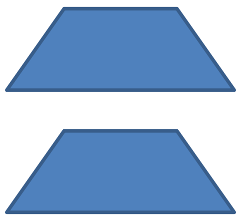
Answers and Quiz

- 4.1 Answers
- 4.1 Quiz

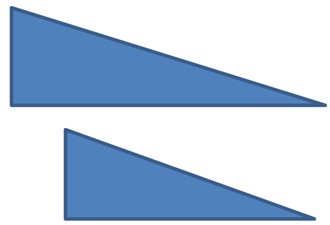
4.2 Apply Congruence and Triangles

Congruent $\cong$

Exactly the same shape and size.

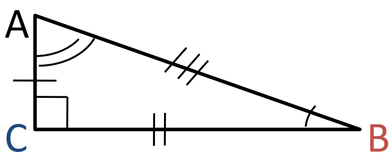


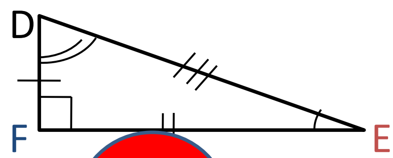
Congruent



Not Congruent

4.2 Apply Congruence and Triangles





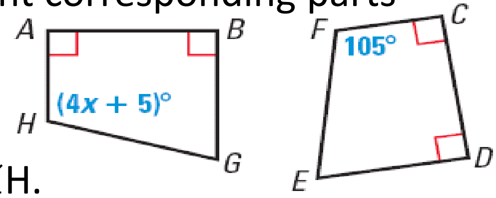
- ΔABC ≅ ΔDEF

ΔABC ≅ ΔEDF

- $\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$
- $\overline{AB} \cong \overline{DE}$ $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$

4.2 Apply Congruence and Triangles

- In the diagram, ABGH ≅ CDEF
  - Identify all the pairs of congruent corresponding parts
  - Find the value of x and find m∠H.

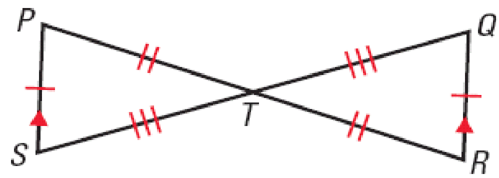


AB ≅ CD, BG ≅ DE, GH ≅ EF, AH ≅ CF
∠A ≅ ∠C, ∠B ≅ ∠D, ∠G ≅ ∠E, ∠H ≅ ∠F

$4x + 5 = 105$
$4x = 100$
$x = 25$
$m\angle H = 105°$

4.2 Apply Congruence and Triangles

- Show that ΔPTS ≅ ΔRTQ



All of the corresponding parts of ΔPTS are congruent to those of ΔRTQ by the indicated markings, the Vertical Angle Theorem and the Alternate Interior Angle theorem.

4.2 Apply Congruence and Triangles

Third Angle Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

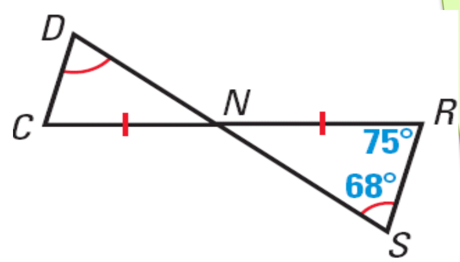
Properties of Congruence of Triangles

Congruence of triangles is Reflexive, Symmetric, and Transitive

75 + 20 + ? = 180
95 + ? = 180
? = 85

4.2 Apply Congruence and Triangles

- In the diagram, what is m∠DCN?



- By the definition of congruence, what additional information is needed to know that ΔNDC ≅ ΔNSR?

m∠DCN = 75°; alt int angle theorem (or 3rd angle theorem)

DN ≅ SN, DC ≅ SR

4.2 Apply Congruence and Triangles

- *228 #4-16 even, 17, 20, 26, 28, 32-40 all = 20 total*

Answers and Quiz

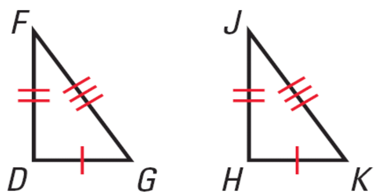
- 4.2 Answers
- 4.2 Quiz

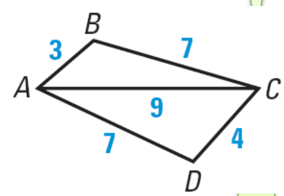
4.3 Prove Triangles Congruent by SSS

SSS (Side-Side-Side Congruence Postulate)

If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent

- True or False
  - $\Delta DFG \cong \Delta HJK$
  - $\Delta ACB \cong \Delta CAD$



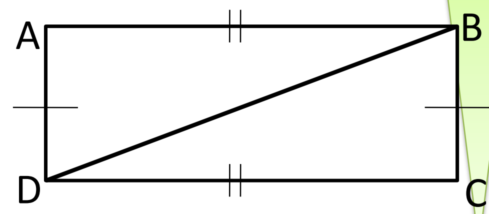


True

False

4.3 Prove Triangles Congruent by SSS

- Given: $\overline{AB} \cong \overline{DC}$; $\overline{AD} \cong \overline{BC}$
- Prove: ΔABD ≅ ΔCDB

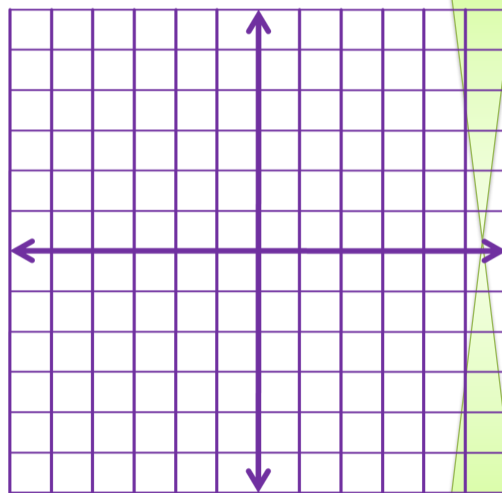


Statements	Reasons

AB ≅ DC; AD ≅ CB (given)
BD ≅ BD (reflexive)
ΔABD ≅ ΔCDB (SSS)

4.3 Prove Triangles Congruent by SSS

ΔJKL has vertices $J(-3, -2)$, $K(0, -2)$, and $L(-3, -8)$. ΔRST has vertices $R(10, 0)$, $S(10, -3)$, and $T(4, 0)$. Graph the triangles in the same coordinate plane and show that they are congruent.

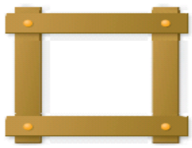
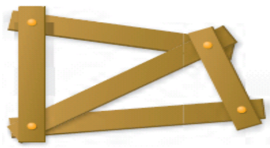
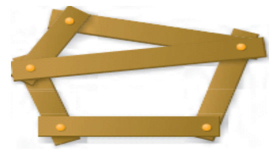


$JK = \sqrt{}((0 - (-3))^2 + (-2 - (-2))^2 = \sqrt{}(9 + 0) = 3$
$KL = \sqrt{}((-3 - 0)^2 + (-8 - (-2))^2 = \sqrt{}(9 + 36) = \sqrt{45}$
$JL = \sqrt{}((-3 - (-3))^2 + (-8 - (-2))^2 = \sqrt{}(0 + 36) = 6$

$RS = \sqrt{}((10 - 10)^2 + (-3 - 0)^2) = \sqrt{}(0 + 9) = 3$
$ST = \sqrt{}((4 - 10)^2 + (0 - (-3))^2) = \sqrt{}(36 + 9) = \sqrt{45}$
$RT = \sqrt{}((4 - 10)^2 + (0 - 0)^2) = \sqrt{}(36 + 0) = 6$

4.3 Prove Triangles Congruent by SSS

- Determine whether the figure is stable.

- *236 #2-30 even, 31-37 all = 22 total*
- *Extra Credit 239 #2, 4 = +2*

Not stable

Stable since has triangular construction

Not stable, lower section does not have triangular construction

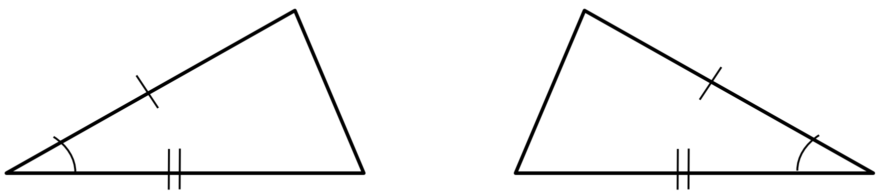
Answers and Quiz

- [4.3 Answers]
- [4.3 Quiz]

4.4 Prove Triangles Congruent by SAS and HL

SAS (Side-Angle-Side Congruence Postulate)

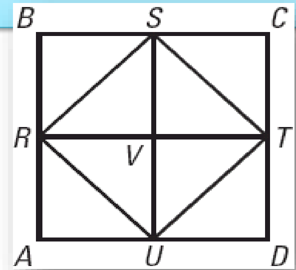
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent



The angle must be between the sides!!!

4.4 Prove Triangles Congruent by SAS and HL

- Given: ABCD is square; R, S, T, and U are midpts; $\overline{RT} \perp \overline{SU}$; $\overline{SV} \cong \overline{VU}$
- Prove: ΔSVR ≅ ΔUVR

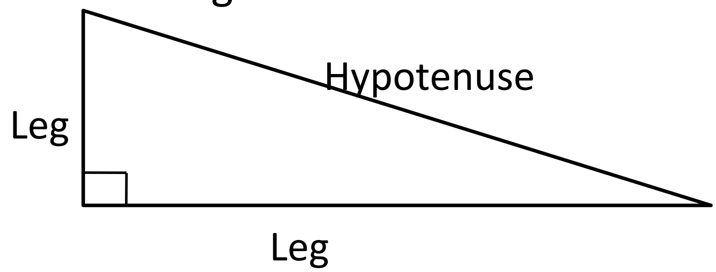


Statements	Reasons

ABCD is square; R, S, T, and U are midpts; RT ⊥ SU; SV ≅ VU (given)
∠SVR and ∠UVR are rt angles (⊥ lines form 4 rt ∠)
∠SVR ≅ ∠UVR (all rt angles are congruent)
RV ≅ RV (reflexive)
ΔSVR ≅ ΔUVR (SAS)

4.4 Prove Triangles Congruent by SAS and HL

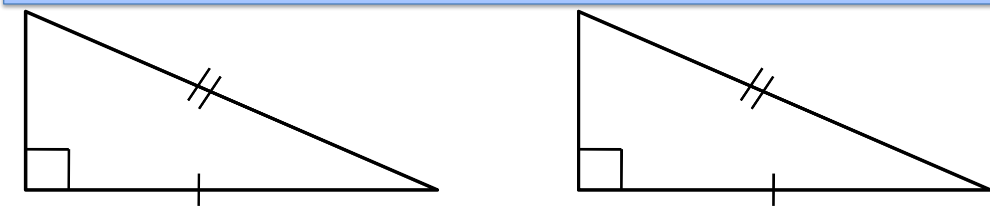
- Right triangles are special
  - If we know two sides are congruent we can use the Pythagorean Theorem (ch 7) to show that the third sides are congruent



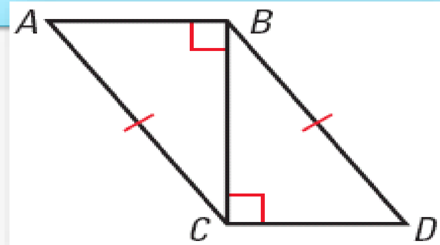
4.4 Prove Triangles Congruent by SAS and HL

HL (Hypotenuse-Leg Congruence Theorem)

If the hypotenuse and a leg of a **right** triangle are congruent to the hypotenuse and a leg of another **right** triangle, then the two triangles are congruent



4.4 Prove Triangles Congruent by SAS and HL



- Given: ∠ABC and ∠BCD are rt ∠s; $\overline{AC} \cong \overline{BD}$
- Prove: ΔACB ≅ ΔDBC

Statements	Reasons

∠ABC and ∠BCD are rt ∠s; AC ≅ BD (given)
ΔACB and ΔDBC are rt Δ (def rt Δ)
BC ≅ CB (reflexive)
ΔACB ≅ ΔDBC (HL)

4.4 Prove Triangles Congruent by SAS and HL

- *243 #4-28 even, 32-48 even = 22 total*

Answers and Quiz

- 4.4 Answers
- 4.4 Quiz

4.5 Prove Triangles Congruent by ASA and AAS

- Use a ruler to draw a line of 5 cm.
- On one end of the line use a protractor to draw a 30° angle.
- On the other end of the line draw a 60° angle.
- Extend the other sides of the angles until they meet.
- Compare your triangle to your neighbor’s.
- This illustrates ASA.

Everyone’s triangle should be congruent

4.5 Prove Triangles Congruent by ASA and AAS

ASA (Angle-Side-Angle Congruence Postulate)

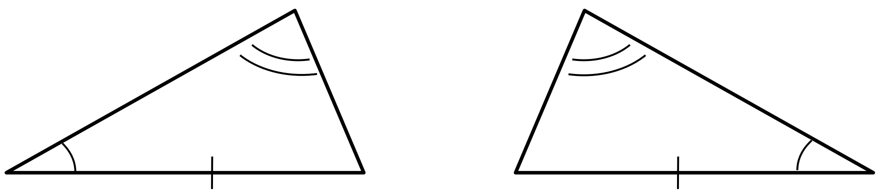
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent

The side must be between the angles!

4.5 Prove Triangles Congruent by ASA and AAS

AAS (Angle-Angle-Side Congruence Theorem)

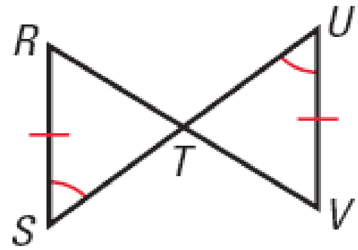
If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle, then the two triangles are congruent



The side is NOT between the angles!

4.5 Prove Triangles Congruent by ASA and AAS

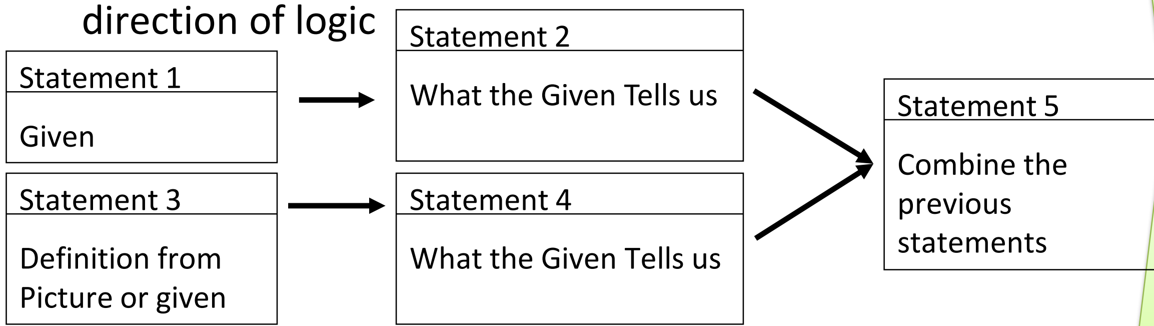
- In the diagram, what postulate or theorem can you use to prove that ΔRST ≅ ΔVUT?



∠RTS ≅ ∠UTV by Vert. Angles are Congruent
ΔRST ≅ ΔVUT by AAS

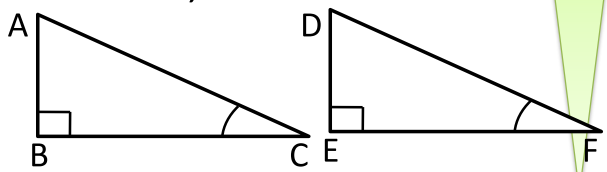
4.5 Prove Triangles Congruent by ASA and AAS

- Flow Proof
  - Put boxes around statements and draw arrows showing direction of logic



4.5 Prove Triangles Congruent by ASA and AAS

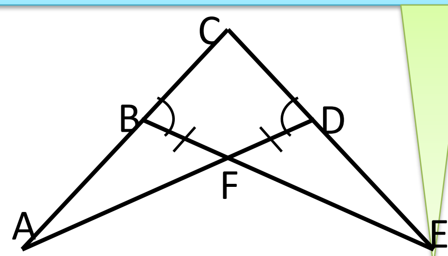
- Given: $\overline{AB} \perp \overline{BC}, \overline{DE} \perp \overline{EF}, \overline{AC} \cong \overline{DF}$, ∠C ≅ ∠F
- Prove: ΔABC ≅ ΔDEF



Statement	Reason
$\overline{AB} \perp \overline{BC}$	Given
∠B is rt ∠	Def ⊥ lines
$\overline{DE} \perp \overline{EF}$	Given
∠E is rt ∠	Def ⊥ lines
∠B ≅ ∠E	Rt ∠s are ≅
$\overline{AC} \cong \overline{DF}$	Given
∠C ≅ ∠F	Given
ΔABC ≅ ΔDEF	AAS

4.5 Prove Triangles Congruent by ASA and AAS

- Given: ∠CBF ≅ ∠CDF, $\overline{BF} \cong \overline{FD}$
- Prove: ΔABF ≅ ΔEDF



Statement	Reason
∠CBF ≅ ∠CDF	Given
∠CBF, ∠ABF supp	Linear Pair Post.
∠CDF, ∠EDF supp	Linear Pair Post.
$\overline{BF} \cong \overline{FD}$	Given
∠ABF ≅ ∠EDF	≅ Supp. Thm.
∠BFA ≅ ∠DFE	Vert. ∠s ≅
ΔABF ≅ ΔEDF	ASA

4.5 Prove Triangles Congruent by ASA and AAS

- *252 #2-20 even, 26, 28, 32-42 even = 18 total*

Answers and Quiz

- [4.5 Answers]
- [4.5 Quiz]

4.6 Use Congruent Triangles

- By the definition of congruent triangles, we know that the corresponding parts have to be congruent

CPCTC

Corresponding Parts of Congruent Triangles are Congruent

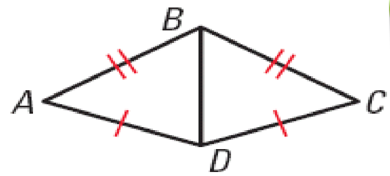
Your book just calls this “definition of congruent triangles”

4.6 Use Congruent Triangles

- To show that parts of triangles are congruent
  - First show that the triangles are congruent using
    - SSS, SAS, ASA, AAS, HL
  - Second say that the corresponding parts are congruent using
    - CPCTC or "def $\cong$ Δ"

4.6 Use Congruent Triangles

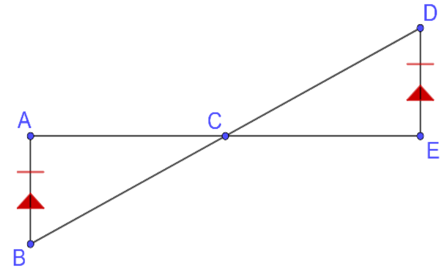
- Write a plan for a proof to show that $\angle A \cong \angle C$



- Show that $\overline{BD} \cong \overline{BD}$ by reflexive
- Show that triangles are $\cong$ by SSS
- Say $\angle A \cong \angle C$ by def $\cong \Delta$ or CPCTC

4.6 Use Congruent Δ

- Given: $\overline{AB} \cong \overline{DE}, \overline{AB} \parallel \overline{DE}$
- Prove: C is the midpoint of $\overline{AB}$



$\overline{AB} \cong \overline{DE}, \overline{AB} \parallel \overline{DE}$		***(given)***
∠B ≅ ∠D, ∠A ≅ ∠E		***(Alt. Int. ∠ Thrm)***
ΔABC ≅ ΔEDC	***(ASA)***	
$\overline{AC} \cong \overline{CE}$		***(CPCTC)***
C is midpoint of $\overline{AE}$		***(Def midpoint)***

4.6 Use Congruent Triangles

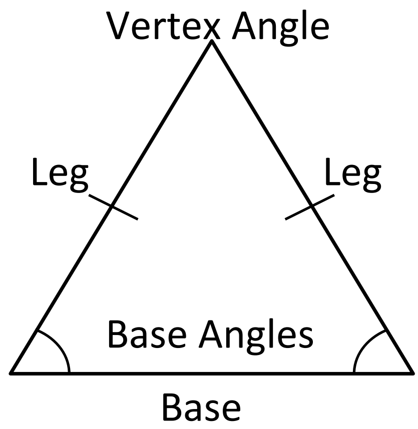
- *259 #2-10 even, 14-28 even, 34, 38, 41-46 all = 21 total*
- *Extra Credit 263 #2, 4 = +2*

Answers and Quiz

- 4.6 Answers
- 4.6 Quiz

4.7 Use Isosceles and Equilateral Triangles

- Parts of an Isosceles Triangle



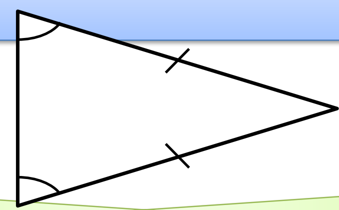
4.7 Use Isosceles and Equilateral Triangles

Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

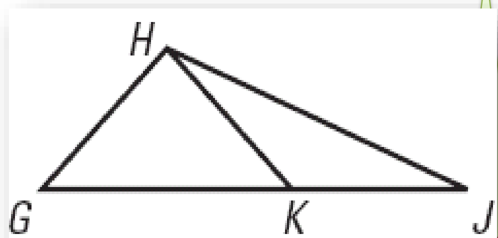
Converse of Base Angles Theorem

If two angles of a triangle are congruent, then the two sides opposite them are congruent.



4.7 Use Isosceles and Equilateral Triangles

- Complete the statement
  - If $\overline{HG} \cong \overline{HK}$, then ∠__?__ ≅ ∠__?__.
  - If ∠KHJ ≅ ∠KJH, then __?__ ≅ __?__.



∠HKG ≅ ∠HGK

KJ ≅ KH

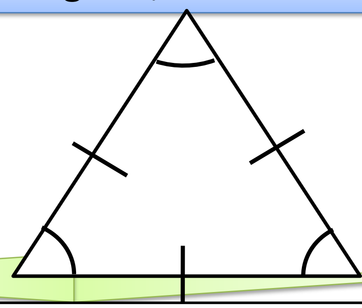
4.7 Use Isosceles and Equilateral Triangles

Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

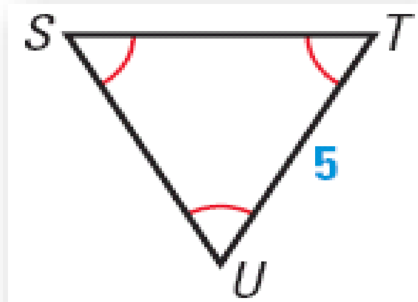
Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is equilateral.



4.7 Use Isosceles and Equilateral Triangles

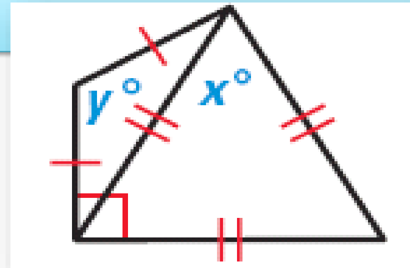
- Find ST
- Find m∠T



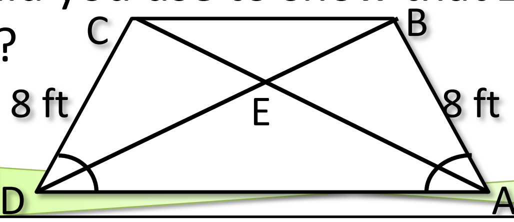
ST = 5
m∠T = 60° (all angles in equilateral/equiangular triangles are 60°)

4.7 Use Isosceles and Equilateral Triangles

- Find the values of x and y



- What triangles would you use to show that ΔAED is isosceles in a proof?



x = 60; equilateral triangle
Each base angle by y; 60 + ? = 90 → ? = 30
Angle sum theorem: 30 + 30 + y = 180 → y = 120

ΔABD, ΔDCA

4.7 Use Isosceles and Equilateral Triangles

- *267 #2-20 even, 24-34 even, 38, 40, 46, 48, 52-60 even = 25 total*

Answers and Quiz

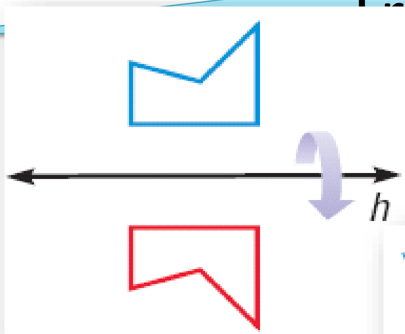
- 4.7 Answers
- 4.7 Quiz

4.8 Perform Congruence Transformations

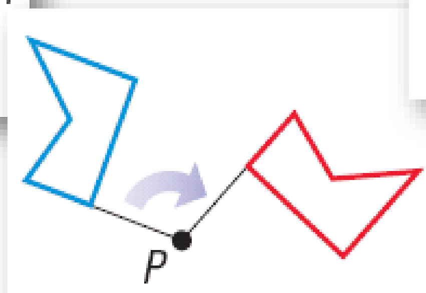
- Transformation is an operation that moves or changes a geometric figure to produce a new figure

- Original figure → Image

4.8 Perform Congruence Transformations



Reflection



Rotation

Translation

4.8 Perform Congruence Transformations

- Name the type of transformation shown.

Reflection

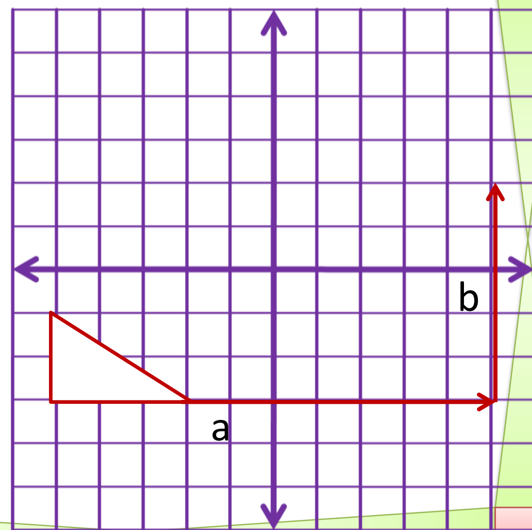
4.8 Perform Congruence Transformations

- Congruence Transformation
  - The shape and size remain the same

  - Translations
  - Rotations
  - Reflections

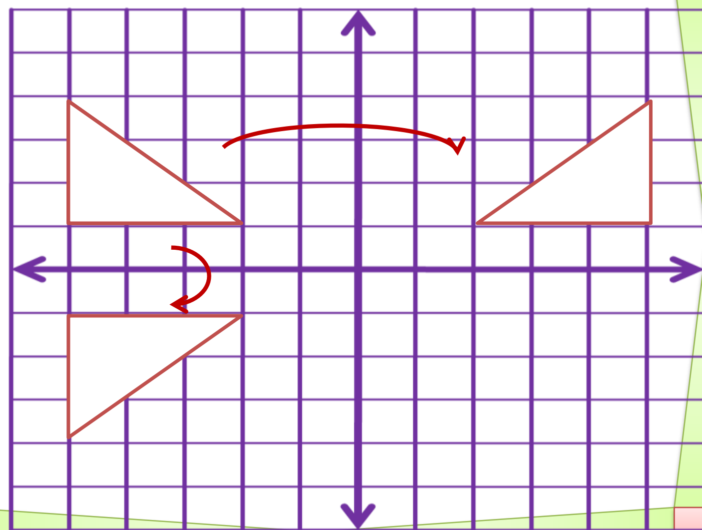
4.8 Perform Congruence Transformations

- Translations
  - Can describe mathematically
  - (x, y) → (x + a, y + b)
  - Moves *a* right, *b* up



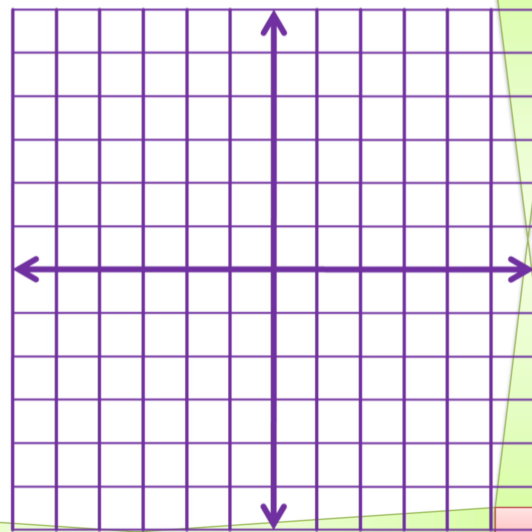
4.8 Perform Congruence Transformations

- Reflections
  - Can be described mathematically by
    - Reflect over y-axis: (x, y) → (-x, y)
    - Reflect over x-axis: (x, y) → (x, -y)



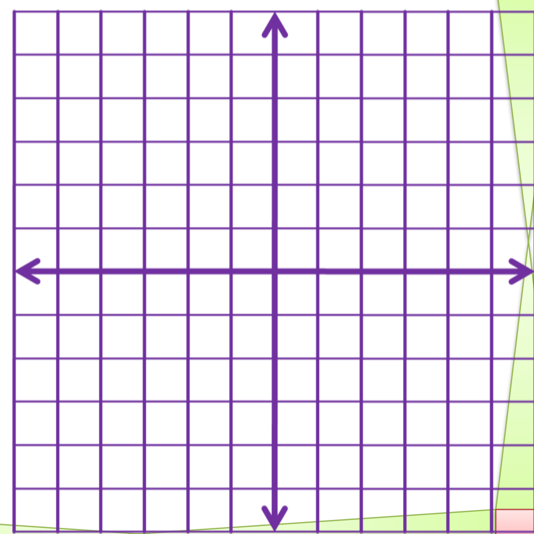
4.8 Perform Congruence Transformations

Figure WXYZ has the vertices W(-1, 2), X(2, 3), Y(5, 0), and Z(1, -1). Sketch WXYZ and its image after the translation (x, y) → (x – 1, y + 3).



4.8 Perform Congruence Transformations

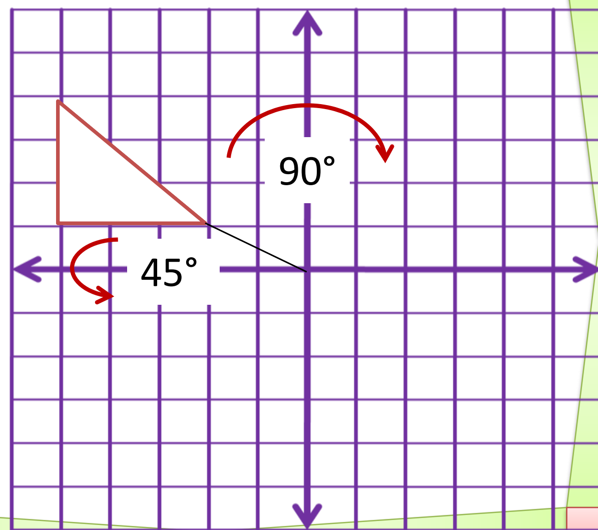
The endpoints of $\overline{RS}$ are R(4, 5) and S(1, -3). A transformation of $\overline{RS}$ results in the image $\overline{TU}$, with coordinates T(4, -5) and U(1, 3). Tell which transformation and write the coordinate rule.



Reflect over x-axis (x, y) → (x, -y)

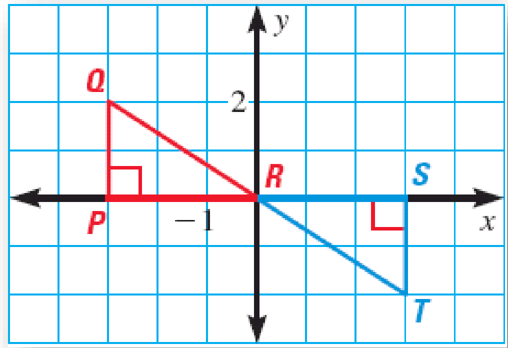
4.8 Perform Congruence Transformations

- Rotations
  - Give center of rotation and degree of rotation
  - Rotations are clockwise or counterclockwise



4.8 Perform Congruence Transformations

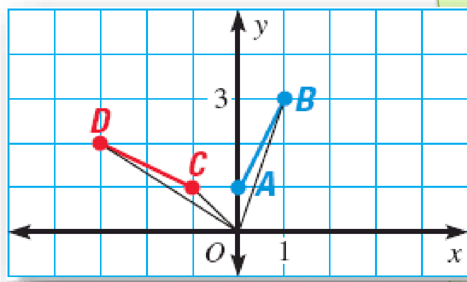
- Tell whether ΔPQR is a rotation of ΔSTR. If so, give the angle and direction of rotation.



180° (either direction is correct)

4.8 Perform Congruence Transformations

- Tell whether ΔOCD is a rotation of ΔOAB. If so, give the angle and direction of rotation



- *276 #2-42 even, 46-50 even = 24 total*
- *Extra Credit 279 #2, 6 = +2*

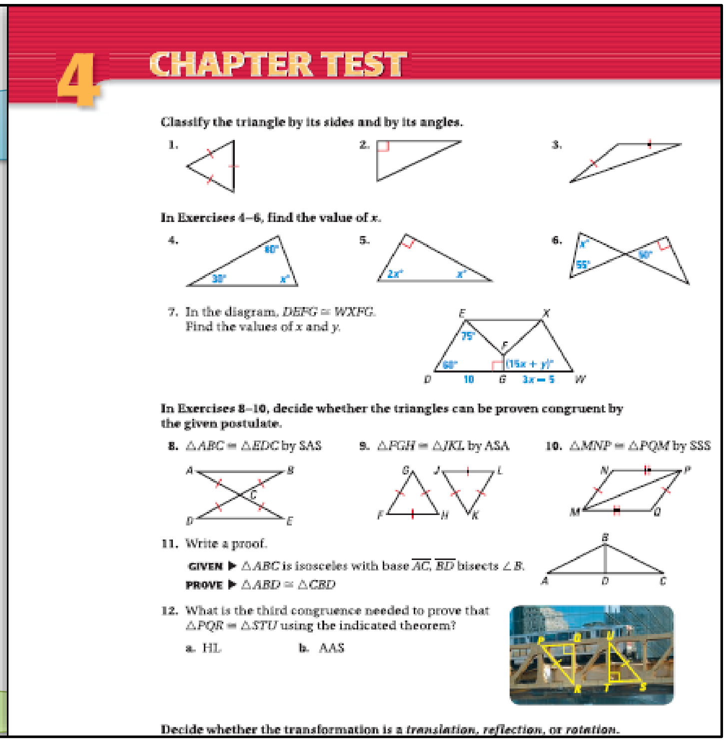
Not a rotation

Answers and Quiz

- 4.8 Answers
- 4.8 Quiz

4.Review

- *286 #1-15*
  = 15 total



4 CHAPTER TEST

Classify the triangle by its sides and by its angles.

1. 2. 3.

In Exercises 4–6, find the value of x.

4. 5. 6.

7. In the diagram, $DEFG \cong WXFG$. Find the values of x and y.

In Exercises 8–10, decide whether the triangles can be proven congruent by the given postulate.

8. $\triangle ABC \cong \triangle EDC$ by SAS
9. $\triangle FGH \cong \triangle JKL$ by ASA
10. $\triangle MNP \cong \triangle PQM$ by SSS

11. Write a proof.

GIVEN ▶ $\triangle ABC$ is isosceles with base $\overline{AC}$, $\overline{BD}$ bisects $\angle B$.

PROVE ▶ $\triangle ABD \cong \triangle CBD$

12. What is the third congruence needed to prove that $\triangle PQR \cong \triangle STU$ using the indicated theorem?

a. HL b. AAS

Decide whether the transformation is a *translation*, *reflection*, or *rotation*.